

in order to use them as arguments in the `mod` function to create the block indicators `blk1` and `blk2` in the two sub-experiments. The block indicator (`blk1`) for the  $2^2$  sub-experiment takes the values 1 and 2. The block indicator (`blk2`) in the  $3^2$  sub-experiment takes the values 1, 2, and 3. The statement `Block <- as.factor((blk1 - 1) * 3 + blk2)` combines the two block indicators into one. When `blk1=1` and `blk2=1`, `Block=1`; when `blk1=1` and `blk2=2`, `Block=2`, and so forth.

Finally, the block indicators are combined with the  $2^2 \times 3^2$  design and the runs are sorted by blocks.

```
> BLMixfac <- cbind(Block,Mixfac)
> BLMixfac <- BLMixfac[order(BLMixfac$Block), ]
```

The first four blocks of the design are shown horizontally below.

```
> BLMixfac
Block A B C D  Block A B C D  Block A B C D  Block A B C D
      1 1 1 2 1      2 1 1 3 1      3 1 1 1 1      4 2 1 2 1
      1 2 2 2 1      2 2 2 3 1      3 2 2 1 1      4 1 2 2 1
      1 1 1 1 2      2 1 1 2 2      3 1 1 3 2      4 2 1 1 2
      1 2 2 1 2      2 2 2 2 2      3 2 2 3 2      4 1 2 1 2
      1 1 1 3 3      2 1 1 1 3      3 1 1 2 3      4 2 1 3 3
      1 2 2 3 3      2 2 2 1 3      3 2 2 2 3      4 1 2 3 3
```

This design will allow estimation of all the terms in the model  $A+B+C+D+A:B+A:D+B:C+B:D$ , but the interactions  $A:B$  and  $C:D$  are confounded with blocks and cannot be estimated.

When the levels of one or more factors in a mixed level factorial are a product of prime powers, and can be represented by the combination of levels of pseudo factors, no interactions among pseudo factors that represent the same factor can be confounded in any sub-experiment. If any interaction among pseudo factors that represents a factor is confounded, then that main effect will also be confounded. For example, consider blocking the 72 combinations of factor levels in a  $3 \times 4 \times 6$  factorial. Factor  $A$  has three levels, factor  $B$  has four levels, and can be represented by all combinations of two two-level pseudo factors  $b_1$  and  $b_2$ , and factor  $C$  has six levels that can be represented by all combinations of a two-level pseudo factor  $c_1$  and a three-level pseudo factor  $c_2$ . Using the pseudo factors, the  $3 \times 4 \times 6$  factorial can be represented by a  $2^3 \times 3^2$  factorial in prime level factors and prime level pseudo factors. The block size must be divisible by the prime numbers 2 and 3 to avoid confounding a main effect, therefore blocks of size 6 or 12 may be possible.

The first sub-experiment is a  $2^3$  composed of two-level pseudo factors  $b_1$ ,  $b_2$ ,  $c_1$  and the second sub-experiment is a  $3^2$  composed of factor  $A$  and pseudo factor  $c_2$ . The first sub-experiment can only be blocked into 2 blocks of 4 in order to avoid confounding the  $b_1 + b_2$  interaction and therefore the  $B$  main